Embedded Functions for Constraints and Variable Strength in Combinatorial Testing

George B. Sherwood

Project objectives and this talk

- Testcover.com service usability enhancement project
  - Simple functions to describe constraints
  - Automatic function evaluation and test case generation
  - Variable strength designs
  - Suitable response times
  - Practical numbers of test cases
- Topics
  - **Calendar constraints:** Conform to constraints with a simple function
  - **Body Mass Index (BMI) report classes:** Use equivalence class functions to reach expected results classes
  - **Shopping cart states:** Simplify complex behavior with functions
  - **Variable strength:** Use hybrid functions for higher strength subarrays while conforming to other constraints

Test model terms

- There are k test factors, e.g. configurations & inputs
- A test case has 1 value for each factor (i.e. a k-tuple)
- A strength-t design ($t \leq k$) includes all required t-tuples of test factor values
- A *partition* includes the allowed combinations (t-tuples) for 1 test case generation instance
- An *equivalence class* includes combinations for 1 class of expected results
- A *constraint* is a condition to specify allowed factor values depending on the values of other factors
- *Masking* occurs when required tuples are missing from test cases for a class of results

Functionally dependent test factor values

- Constraints can be described using functionally dependent test factor values
- Functional dependence:
  1 or more values of a dependent factor are identified by other, determinant factors
  Determinant factors' values → dependent factor values
- Example: The last day of any month is identified its month and year
  Month, Year → Last day values
  ℓ = number of determinant factors (ℓ = 2 in this example)
- Use Direct Product Block (DPB) notation with or without embedded combination functions

Direct Product Block (DPB) notation

Fixed values form

```
Calendar Example without last_day function
Month
Day
Year
#ok All good dates
jan feb mar apr may jun jul aug sep oct nov dec
1 10
2015 2016 2017
+ long month last day
jan mar may jul aug oct dec
31
2015 2016 2017
+ short month last day
apr jun sep nov
30
2015 2016 2017
+ feb last day
feb
28
2015 2017
+ leap day
feb
29
2016
```

- Valid calendar dates example with boundary checking
- Factor values are on separate lines
- All combinations in a block are allowed
- Partition of multiple blocks includes union of their allowed combinations

Direct Product Block (DPB) notation

Fixed values form

```
Calendar Example without last_day function
Month
Day
Year
#ok All good dates
jan feb mar apr may jun jul aug sep oct nov dec
1 10
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2015 2016 2017
+ feb last day
feb
28
2015 2017
+ leap day
feb
29
2016
```

Functionally dependent form

```
Calendar Example with last_day function
$month
Day
$year
#ok All good dates
jan feb mar apr may jun jul aug sep oct nov dec
1 10 last_day($month,$year)
2015 2016 2017
```

- Month, Year → Last day values
- Factors renamed as variables for function arguments:

  $month $year
- Day values:

  1 10 last_day($month,$year)
- 5 blocks now represented by only 1 block

Evaluation of calendar last_day function

Fixed values

+ Day: 1 10
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 31
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 30
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 28
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 29
… … … … … … …
… … … … … … …
… … … … … … …

5 FV blocks

Test cases
1 … … … … … … …
2 … … … … … … …
3 … … … … … … …
.. … … … … … … …
.. … … … … … … …
.. … … … … … … …

Functionally dependent

+ FD block
… … … … … … …
… … cf_1 … … … …
… … … … … … …

All fixed values 1 10 last_day($month,$year)

FV: 1 10 Evaluation & collection

cf_1: last_day($month,$year)

+ Day: 1 10
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 31
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 30
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 28
… … … … … … …
… … … … … … …
… … … … … … …

+ Day: 29
… … … … … … …
… … … … … … …
… … … … … … …

5 FV blocks

Test cases
1 … … … … … … …
2 … … … … … … …
3 … … … … … … …
.. … … … … … … …
.. … … … … … … …
.. … … … … … … …

Equivalence class functions

- Equivalence class functions identify classes of expected results
- Adult_report($Age,$Weight,$Height) returns report class:

  underweight normal overweight obese no
- Combinations of determinant values are required
- An equivalence class factor has all its class values

  because all of its determinant combinations are evaluated
- Pairwise test case generation covers all of these classes

  because they are values of the equivalence class factor
- Equivalence classes are paired with nondeterminant factors
- Higher strength not needed to reach well defined classes

Equivalence class factors

Functionally dependent form

```
Body Mass Index Report Application - 1 partition
with EC factors
$Age
$Weight
$Height
$Sex
Intake
Medicare equivalence class
Child equivalence class
Adult equivalence class
#
19 42 67
131 180
64 71
female male
2000 3000
Medicare_report($Age)
Child_report($Age,$Sex)
Adult_report($Age,$Weight,$Height)
```

- 5 input factors
- 3 equivalence class factors

  Medicare_report

    $Age ≥ 65: yes

    $Age < 65: no

  Child_report

    $Age ≥ 20: no

    $Sex = female: girl

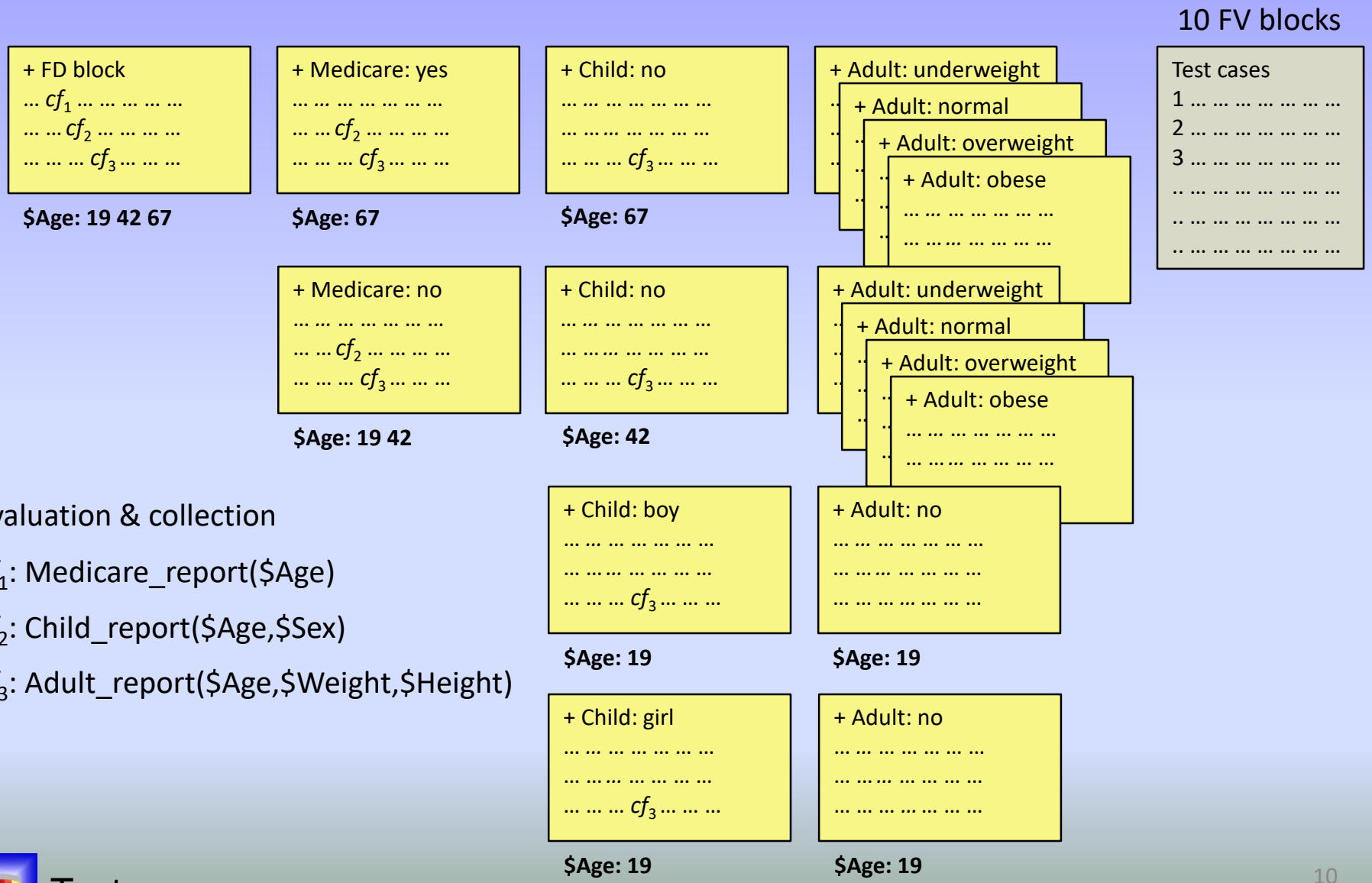
    $Sex = male: boy

  Adult_report

    $Age < 20: no

    else: BMI classification

Evaluation of equivalence class functions



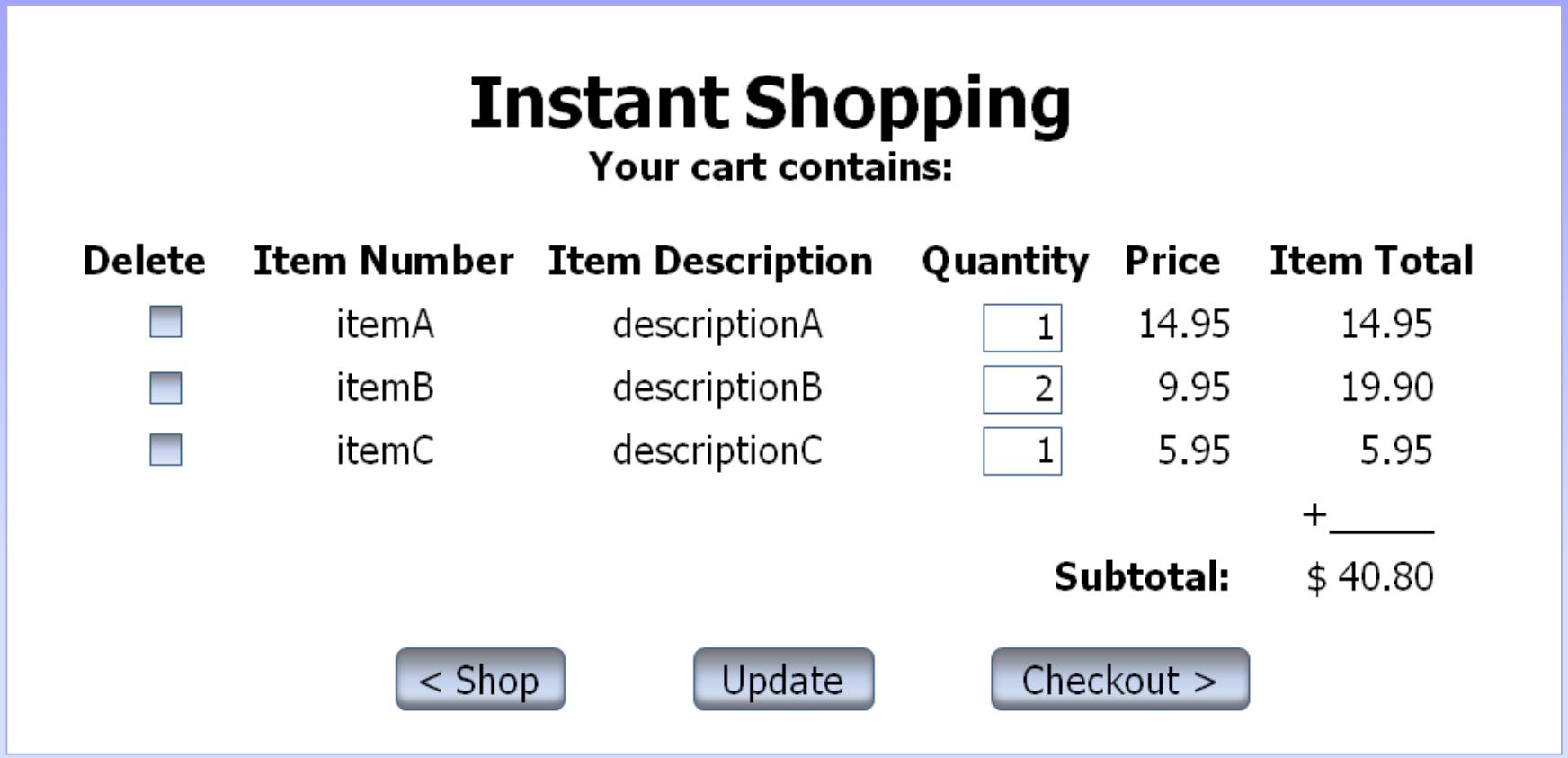
Evaluation & collection

cf_1: Medicare_report($Age)

cf_2: Child_report($Age,$Sex)

cf_3: Adult_report($Age,$Weight,$Height)

Constraint simplification



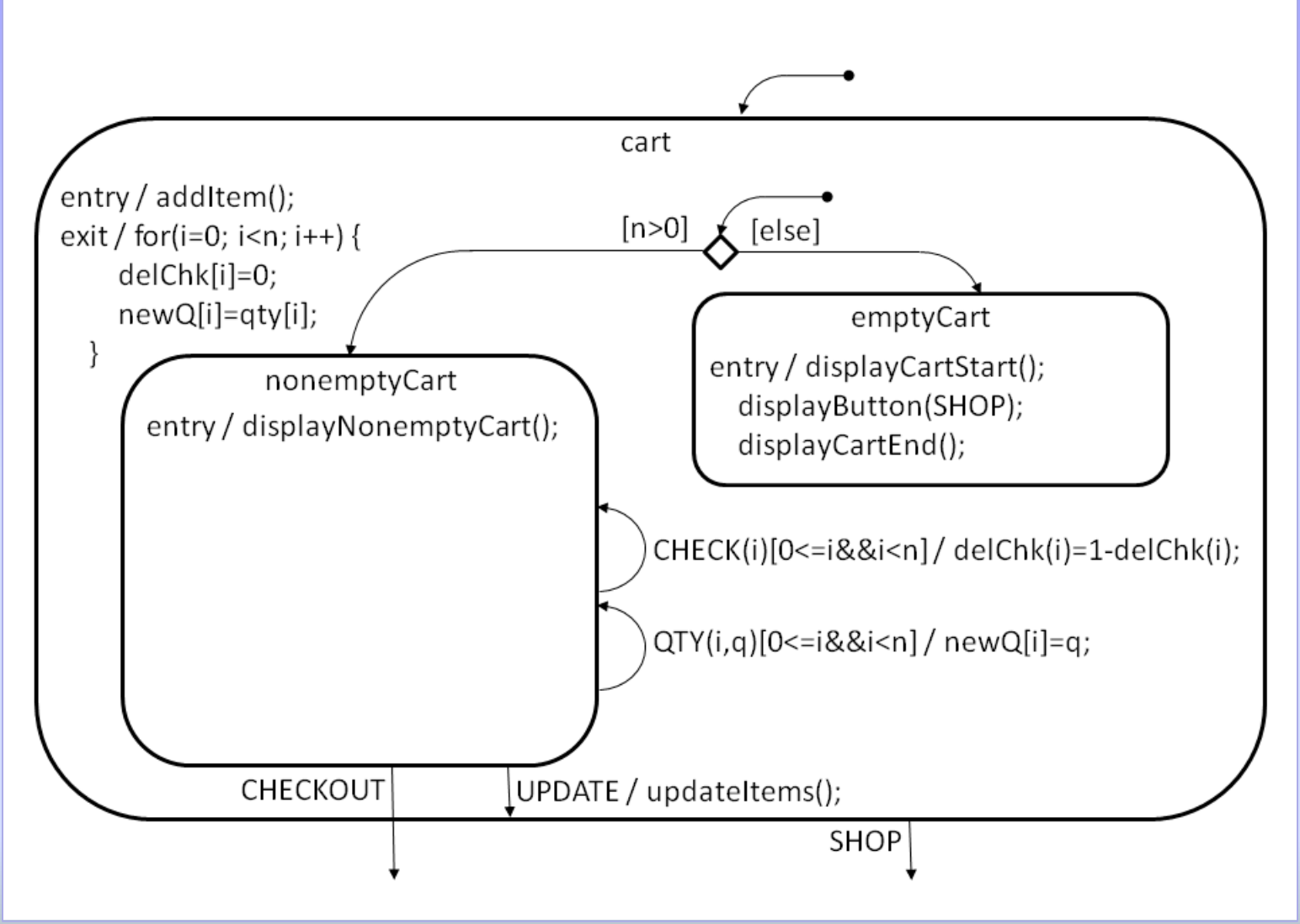
Constraints:

- Items in different positions must be different
- Factor values may be NULL (unused)
- Equivalence classes are target UML leaf states to avoid masking

Simplification for nonemptyCart to nonemptyCart transition:

- Without combination functions: 33 blocks
- With 9 combination functions: 3 blocks
- Average function length: 7 lines

Shopping cart state diagram



Shopping cart test factors

- Program variables
  - Indexed for cart position
    - $delChk[0]
    - $item[0]
    - $qty[0]
    - $newQ[0]
- Current state
  - nonemptyCart
- Event (trigger)
  - CHECK QTY UPDATE

Test Factor	Test Factor Values	Combination Functions	Indication
$newItem	NULL		Item to place in cart
$n	1 2 3		Number of items in cart
$delChk[0]	0 1	f_delChk_CQ f_delChk_U	Delete box checked in cart position 0
$item[0]	itemA itemB itemC	f_item	Item in cart position 0
$qty[0]	1 2 10	f_qty	Quantity of item in cart position 0
$newQ[0]	0 1 2 10	f_newQ_CQ f_newQ_U	New quantity shown in cart position 0
$delChk[1]	0 1 NULL	f_delChk_CQ f_delChk_U	Delete box checked in cart position 1
$item[1]	itemA itemB itemC NULL	f_item	Item in cart position 1
$qty[1]	1 2 10 NULL	f_qty	Quantity of item in cart position 1
$newQ[1]	0 1 2 10 NULL	f_newQ_CQ f_newQ_U	New quantity shown in cart position 1
$delChk[2]	0 1 NULL	f_delChk_CQ f_delChk_U	Delete box checked in cart position 2
$item[2]	itemA itemB itemC NULL	f_item	Item in cart position 2
$qty[2]	1 2 10 NULL	f_qty	Quantity of item in cart position 2
$newQ[2]	0 1 2 10 NULL	f_newQ_CQ f_newQ_U	New quantity shown in cart position 2
$i	0 1 2 NULL	f_i	Cart position for event
$q	0 1 2 10 NULL		Quantity for event
state	nonemptyCart		Source state
event	CHECK($i) QTY($i,$q) UPDATE	f_event_CHECK f_event_QTY	Trigger to target state

Shopping cart blocks

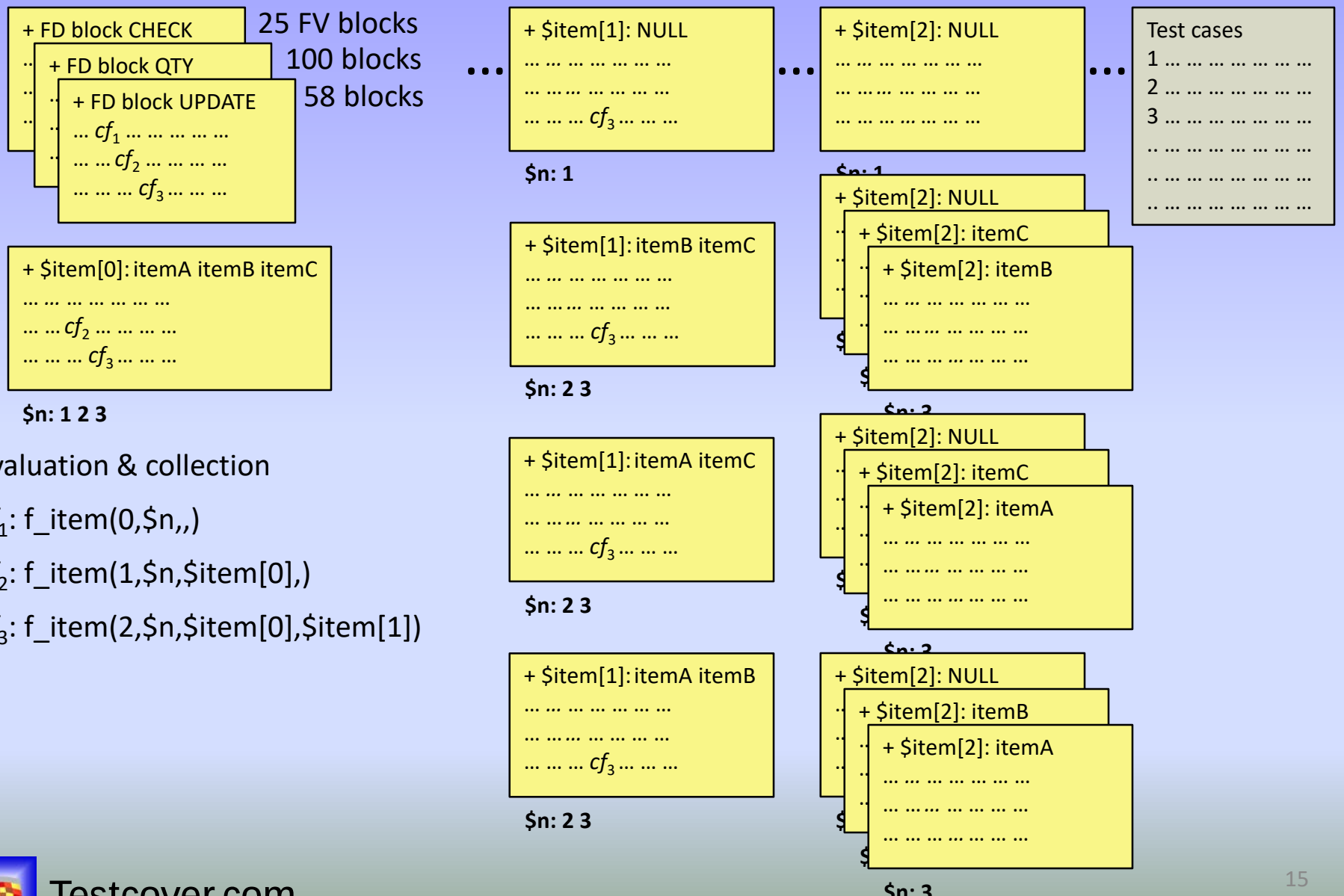
- DPB request in FD form
- 18 factor names
- Values in 3 blocks
- Allowed item values: itemA itemB itemC NULL
- f_item function returns item values in all 3 blocks for:
  - $item[0]
  - $item[1]
  - $item[2]

Shopping Cart - transition design - SCA (2,18)
$newItem
$n
$delChk[0]
$item[0]
$qty[0]
$newQ[0]
$delChk[1]
$item[1]
$qty[1]
$newQ[1]
$delChk[2]
$item[2]
$qty[2]
$newQ[2]
$i
$q
State
Event
#NN nonemptyCart to nonemptyCart
+ nonemptyCart to nonemptyCart; CHECK
NULL
1 2 3
f_delChk_CQ(0,$n)
f_item(0,$n,,)
f_qty(0,$n)
f_newQ_CQ(0,$n)
f_delChk_CQ(1,$n)
f_item(1,$n,$item[0],)
f_qty(1,$n)
f_newQ_CQ(1,$n)
f_delChk_CQ(2,$n)
f_item(2,$n,$item[0],$item[1])
f_qty(2,$n)
f_newQ_CQ(2,$n)
f_i($n)
NULL
nonemptyCart
f_event_CHECK($i)

+ nonemptyCart to nonemptyCart; QTY
NULL
1 2 3
f_delChk_CQ(0,$n)
f_item(0,$n,,)
f_qty(0,$n)
f_newQ_CQ(0,$n)
f_delChk_CQ(1,$n)
f_item(1,$n,$item[0],)
f_qty(1,$n)
f_newQ_CQ(1,$n)
f_delChk_CQ(2,$n)
f_item(2,$n,$item[0],$item[1])
f_qty(2,$n)
f_newQ_CQ(2,$n)
f_i($n)
0 1 2 10
nonemptyCart
f_event_QTY($i,$q)

+ nonemptyCart to nonemptyCart; UPDATE
NULL
1 2 3
f_delChk_U(0,$n,1,0,1,0)
f_item(0,$n,,)
f_qty(0,$n)
f_newQ_U(0,$n,1,0,1,0)
f_delChk_U(1,$n,$delChk[0],$newQ[0],1,0)
f_item(1,$n,$item[0],)
f_qty(1,$n)
f_newQ_U(1,$n,$delChk[0],$newQ[0],1,0)
f_delChk_U(2,$n,$delChk[0],$newQ[0],$delChk[1],$newQ[1])
f_item(2,$n,$item[0],$item[1])
f_qty(2,$n)
f_newQ_U(2,$n,$delChk[0],$newQ[0],$delChk[1],$newQ[1])
NULL
NULL
nonemptyCart
UPDATE

Evaluation of composite functions



Evaluation & collection

cf_1: f_item(0,$n,,)

cf_2: f_item(1,$n,$item[0],)

cf_3: f_item(2,$n,$item[0],$item[1])

Hybrid functions for variable strength

- Hybrid functions return tuples of their arguments

  pair($a,$b) returns ($a,$b) triple($a,$b,$c) returns ($a,$b,$c)
- Hybrid factors force higher strength in pairwise designs

  Factors ($a,$b) & ($c,$d) force strength 4 coverage of factors $a $b $c $d
- Hybrid factors were defined for variable strengths ≤ 6
- Hybrid functions inherit values from their superblocks

  So they conform to other test model constraints if present
- Variable strength test cases were generated for the shopping cart example, e.g. 2 triple factors were used for strength 6:

  ($delChk[0],$newQ[0],$delChk[1]) ($newQ[1],$delChk[2],$newQ[2])

Conclusions

- Test designs can conform to constraints described by simple functions in a familiar language
- Embedded functions results are equivalent to using fixed values
- Equivalence class functions can eliminate the need for higher strength generation to reach classes of expected results
- Embedded functions can simplify testing complex systems through encapsulation and composite relations
- Hybrid functions can increase strength among selected test factors while conforming to system constraints

Next steps

- Substitution function implementation
- Interface to include and manage functions
- Additional resource management
- Security considerations
- More testing