Functional Dependence and Equivalence Class Factors in Combinatorial Test Designs

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This talk is about:

- Evaluation of test design coverage before test case generation
  - *Partition* includes test case generation combinations
  - *Equivalence class* includes expected result combinations
  - Typically a partition spans multiple equivalence classes
  - Formulas for equivalence class coverage
- Design choices to improve coverage
  - Use increased generation strength ***t***
  - Align partitions with equivalence classes as needed

Body mass index report requirements

R1. Input data for patient database table:

- Age in years
- Weight in pounds
- Height in inches
- Sex (female, male)
- Intake in kilocalories per day

R2. Compute & store body mass index:

- BMI = 703 x Weight / Height^2

R3. Age ≥ 65: Generate Medicare report

Body mass index report requirements

continued

R4. Age < 20 Generate Child report:

Girl, percentile from female BMI-age table

Boy, percentile from male BMI-age table

R5. Age ≥ 20 Generate Adult report:

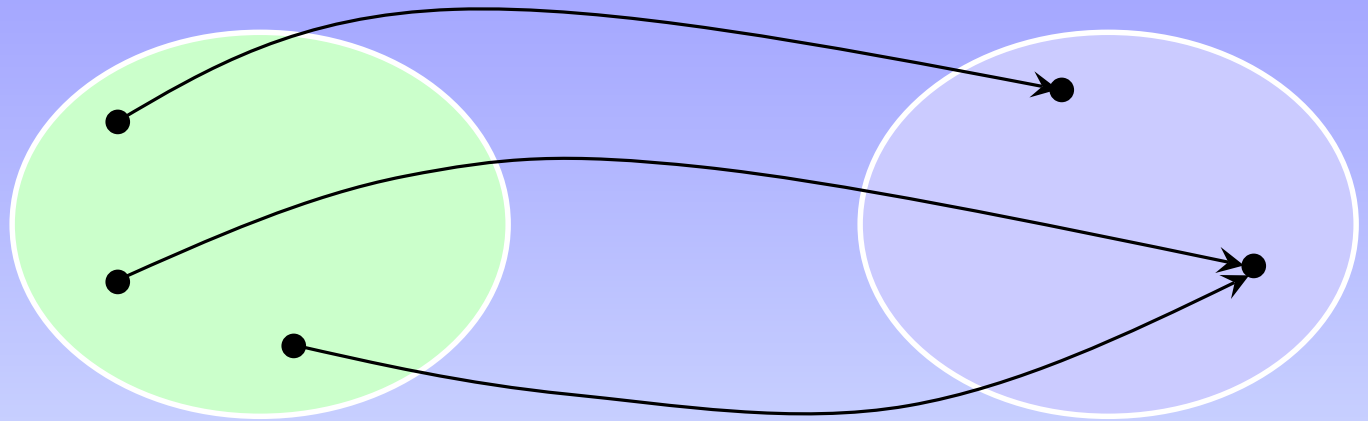
Underweight, BMI < 18.5

Normal, 18.5 ≤ BMI < 25.0

Overweight, 25.0 ≤ BMI < 30.0

Obese, 30.0 ≤ BMI

Functional dependence



Mapping from a determinant set to one element in a dependent set

Examples:

- $y = \sin(x)$
- US postal address index → ZIP code
- ℓ-tuple → m-tuple
- BMI = 703 x Weight / Height^2
- determinant factor values → functionally dependent factor values

Functionally dependent test factors

- A test factor is functionally dependent when its value is identified by ℓ determinant factors

  Example:

  BMI is determined by ℓ = 2 factors, Weight & Height

  Weight, Height → BMI

- Independent factors which are not part of this relation are nondeterminant

  Example:

  n = 3 nondeterminant factors, Age, Sex & Intake

Equivalence classes

- Equivalence classes group test factor combinations by similar expected results

  Example:

  Age ≤ 0 leads to invalid, error handling class

- Classes help insure test design coverage

  Example:

  The Medicare, Child and Adult reports each have multiple, valid equivalence classes

Report	Valid equivalence classes				
Medicare	no	yes			
Child	no	girl	boy		
Adult	no	underweight	normal	overweight	obese

Equivalence classes are functionally dependent

Input, configuration values → result

→ equivalence class

Equivalence class factor examples:	ℓ	n
Age → Medicare classes	1	4
Age, Sex → Child classes	2	3
Age, Weight, Height → Adult classes	3	2

Equivalence class factor values							
Factors for test case generation					Factors determined from test cases		
Independent input factors					*Functionally dependent factors*		
Age	***Weight***	***Height***	***Sex***	***Intake***	***Medicare***	***Child***	***Adult***
19	131	64	female	2000	no	girl	no
67	131	64	male	3000	yes	no	normal

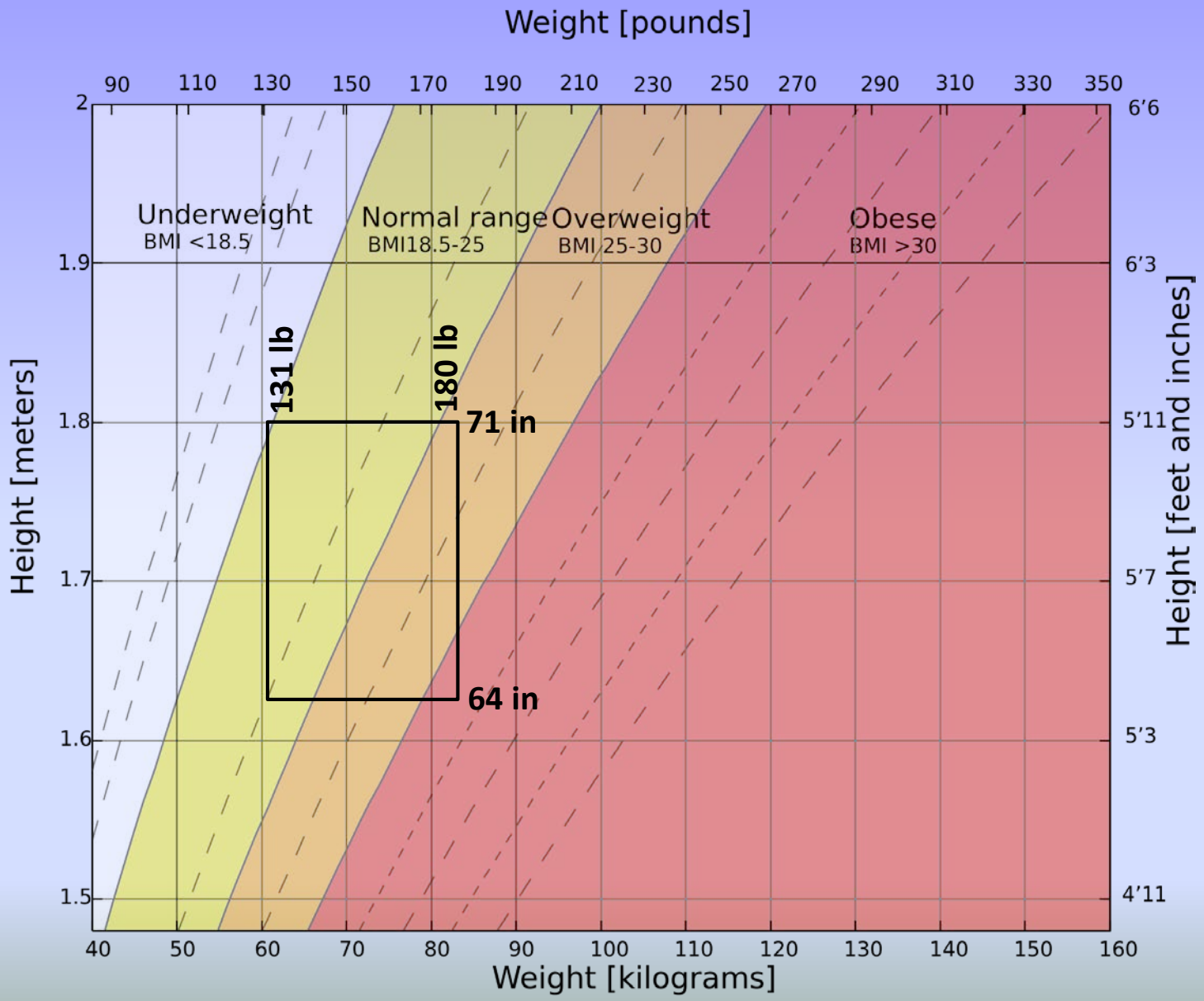
Determinant coverage

Combinations to reach equivalence classes – Medicare							
Input factors					Equivalence class factors		
Determinant	*Nondeterminant*				*Functionally dependent*		
Age	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*
42					no		
67					yes		

Combinations to reach equivalence classes – Child							
Input factors					Equivalence class factors		
Determinant	*Nondeterminant*		*Determinant*	*Non-determinant*		*Functionally dependent*	
Age	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*
19			female			girl	
19			male			boy	
42 or 67						no	

Combinations to reach equivalence classes – Adult							
Input factors					Equivalence class factors		
Determinant			*Nondeterminant*				*Functionally dependent*
Age	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*
19							no
42 or 67	131	64					normal
42 or 67	131	71					underweight
42 or 67	180	64					obese
42 or 67	180	71					overweight

Adult report classes



Nondeterminant coverage

- Nondeterminant factors provide coverage unrelated to the functional dependence
- A nondeterminant subarray $\mathcal{J}$ consists of a functionally dependent factor and its n nondeterminant factors.

  Example:

Nondeterminant subarray $\mathcal{J}$ – Child							
Input factors					Equivalence class factors		
Determinant	*Nondeterminant*		*Determinant*	*Non-determinant*		*Functionally dependent*	
Age	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*

- $\mathcal{J}$ has *nondeterminant strength* s when every subarray of s factors *including the functionally dependent factor* has every s-tuple in at least one test case.

  Example: When s = 3 each pair of nondeterminant factor values is associated with each Child equivalence class.

Nondeterminant strength formula

$$s = t - \ell + 1$$

Test cases from one strength-4 partition for multiple equivalence classes – Child									
Input factors						Equivalence class factors			An expected result
	Determinant	*Nondeterminant*		*Determinant*	*Non-determinant*		*Functionally dependent*		
Test Case	*Age*	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*	*BMI*
1	19	131	64	female	2000	no	girl	no	22.5
2	19	131	64	male	3000	no	boy	no	22.5
3	19	131	71	female	3000	no	girl	no	18.3
4	19	131	71	male	2000	no	boy	no	18.3
5	19	180	64	female	3000	no	girl	no	30.9
6	19	180	64	male	2000	no	boy	no	30.9
7	19	180	71	female	2000	no	girl	no	25.1
8	19	180	71	male	3000	no	boy	no	25.1
9	42	131	64	female	3000	no	no	normal	22.5
…									

Age, Sex → Child classes

$s = t - \ell + 1 = 4 - 2 + 1 = 3$.

All pairs of nondeterminant factor values are associated with each Child class.

$s \leq n + 1$ because there are $n + 1$ columns in the subarray.

Coverage formulas

Partition contains	Strength to cover equivalence class(es)	Nondeterminant strength
multiple values for an equivalence class factor	$t \geq \ell$	$s = t - \ell + 1, \quad t \leq \ell + n$ $s = n + 1, \quad t \geq \ell + n$
one value for an equivalence class factor	any	$s = t + 1, \quad t \leq n$ $s = n + 1, \quad t \geq n$

Multiple equivalence classes – Coverage depends on ℓ

Example: $t \geq 3$ to reach all classes; choose $t = 3$

Medicare classes: $\ell = 1$; $s = 3$

Child classes: $\ell = 2$; $s = 2$

Adult classes: $\ell = 3$; $s = 1$ (coverage problem)

One equivalence class – Coverage is independent of ℓ

Example: any strength to reach *one* class; choose $t = 2$

$s = 3$ for each class in its own partition

One strength-3 partition for multiple equivalence classes – Medicare

Input factors						Equivalence class factors			An expected result
	Determinant	*Nondeterminant*				*Functionally dependent*			
Test Case	*Age*	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*	*BMI*
1	19	131	64	female	2000	no	girl	no	22.5
2	19	131	64	male	3000	no	boy	no	22.5
3	19	131	71	male	2000	no	boy	no	18.3
4	19	180	64	female	3000	no	girl	no	30.9
5	19	180	71	female	2000	no	girl	no	25.1
6	19	180	71	male	3000	no	boy	no	25.1
7	42	131	64	female	2000	no	no	normal	22.5
8	42	131	64	male	3000	no	no	normal	22.5
9	42	131	71	female	3000	no	no	underweight	18.3
10	42	180	64	male	2000	no	no	obese	30.9
11	42	180	71	female	2000	no	no	overweight	25.1
12	42	180	71	male	3000	no	no	overweight	25.1
13	67	131	64	female	2000	yes	no	normal	22.5
14	67	131	64	male	3000	yes	no	normal	22.5
15	67	131	71	female	3000	yes	no	underweight	18.3
16	67	180	64	male	2000	yes	no	obese	30.9
17	67	180	71	female	2000	yes	no	overweight	25.1
18	67	180	71	male	3000	yes	no	overweight	25.1

Age → Medicare classes

$s = t - \ell + 1 = 3 - 1 + 1 = 3$.

All pairs of nondeterminant factor values are associated with each Medicare class.

One strength-3 partition for multiple equivalence classes – Child

Input factors						Equivalence class factors			An expected result
	Determinant	*Nondeterminant*		*Determinant*	*Non-determinant*		*Functionally dependent*		
Test Case	*Age*	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*	*BMI*
1	19	131	64	female	2000	no	girl	no	22.5
2	19	131	64	male	3000	no	boy	no	22.5
3	19	131	71	male	2000	no	boy	no	18.3
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6	19	180	71	male	3000	no	boy	no	25.1
7	42	131	64	female	2000	no	no	normal	22.5
8	42	131	64	male	3000	no	no	normal	22.5
9	42	131	71	female	3000	no	no	underweight	18.3
10	42	180	64	male	2000	no	no	obese	30.9
11	42	180	71	female	2000	no	no	overweight	25.1
12	42	180	71	male	3000	no	no	overweight	25.1
13	67	131	64	female	2000	yes	no	normal	22.5
14	67	131	64	male	3000	yes	no	normal	22.5
15	67	131	71	female	3000	yes	no	underweight	18.3
16	67	180	64	male	2000	yes	no	obese	30.9
17	67	180	71	female	2000	yes	no	overweight	25.1
18	67	180	71	male	3000	yes	no	overweight	25.1

Age, Sex → Child classes

$s = t - \ell + 1 = 3 - 2 + 1 = 2$.

All nondeterminant factor values are associated with each Child class.

One strength-3 partition for multiple equivalence classes – Adult

Input factors						Equivalence class factors			An expected result
	Determinant			*Nondeterminant*				*Functionally dependent*	
Test Case	*Age*	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*	*BMI*
1	19	131	64	female	2000	no	girl	no	22.5
2	19	131	64	male	3000	no	boy	no	22.5
3	19	131	71	male	2000	no	boy	no	18.3
4	19	180	64	female	3000	no	girl	no	30.9
5	19	180	71	female	2000	no	girl	no	25.1
6	19	180	71	male	3000	no	boy	no	25.1
7	42	131	64	female	2000	no	no	normal	22.5
8	42	131	64	male	3000	no	no	normal	22.5
9	42	131	71	female	3000	no	no	underweight	18.3
10	42	180	64	male	2000	no	no	obese	30.9
11	42	180	71	female	2000	no	no	overweight	25.1
12	42	180	71	male	3000	no	no	overweight	25.1
13	67	131	64	female	2000	yes	no	normal	22.5
14	67	131	64	male	3000	yes	no	normal	22.5
15	67	131	71	female	3000	yes	no	underweight	18.3
16	67	180	64	male	2000	yes	no	obese	30.9
17	67	180	71	female	2000	yes	no	overweight	25.1
18	67	180	71	male	3000	yes	no	overweight	25.1

Age, Weight, Height → Adult classes

$s = t - \ell + 1 = 3 - 3 + 1 = 1$.

Some nondeterminant factor values might not be associated with each Adult class.

Neither male nor 2000 is associated with the underweight class.

One strength-4 partition for multiple equivalence classes – Adult

Input factors						Equivalence class factors			An expected result
	Determinant			*Nondeterminant*				*Functionally dependent*	
Test Case	*Age*	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*	*BMI*
1	19	131	64	female	2000	no	girl	no	22.5
2	19	131	64	male	3000	no	boy	no	22.5
3	19	131	71	female	3000	no	girl	no	18.3
4	19	131	71	male	2000	no	boy	no	18.3
5	19	180	64	female	3000	no	girl	no	30.9
6	19	180	64	male	2000	no	boy	no	30.9
7	19	180	71	female	2000	no	girl	no	25.1
8	19	180	71	male	3000	no	boy	no	25.1
9	42	131	64	female	3000	no	no	normal	22.5
10	42	131	64	male	2000	no	no	normal	22.5
11	42	131	71	female	2000	no	no	underweight	18.3
12	42	131	71	male	3000	no	no	underweight	18.3
13	42	180	64	female	2000	no	no	obese	30.9
14	42	180	64	male	3000	no	no	obese	30.9
15	42	180	71	female	3000	no	no	overweight	25.1
16	42	180	71	male	2000	no	no	overweight	25.1
17	67	131	64	female	2000	yes	no	normal	22.5
18	67	131	64	male	3000	yes	no	normal	22.5
19	67	131	71	female	3000	yes	no	underweight	18.3
20	67	131	71	male	2000	yes	no	underweight	18.3
21	67	180	64	female	3000	yes	no	obese	30.9
22	67	180	64	male	2000	yes	no	obese	30.9
23	67	180	71	female	2000	yes	no	overweight	25.1
24	67	180	71	male	3000	yes	no	overweight	25.1

Age, Weight, Height → Adult classes

$s = t - \ell + 1 = 4 - 3 + 1 = 2$.

All nondeterminant factor values are associated with each Adult class.

Testcover.com

One strength-2 partition for one equivalence class – Adult underweight									
Input factors						Equivalence class factors			An expected result
	Determinant			*Nondeterminant*				*Functionally dependent*	
Test Case	*Age*	*Weight*	*Height*	*Sex*	*Intake*	*Medicare*	*Child*	*Adult*	*BMI*
uw1	65	118	72	male	3000	yes	no	underweight	16.0
uw2	65	128	70	female	2000	yes	no	underweight	18.4
uw3	64	128	72	male	2000	no	no	underweight	17.4
uw4	20	118	70	male	2000	no	no	underweight	16.9
uw5	64	118	70	female	3000	no	no	underweight	16.9
uw6	20	128	72	female	3000	no	no	underweight	17.4

One partition for each Adult underweight class:

Age, Weight, Height → Adult classes

$s = t + 1 = 2 + 1 = 3$.

All pairs of nondeterminant factor values are associated with each Adult class.

One partition for both Medicare classes:

Age → Medicare classes

$s = t - \ell + 1 = 2 - 1 + 1 = 2$.

All nondeterminant factor values are associated with each Medicare class.

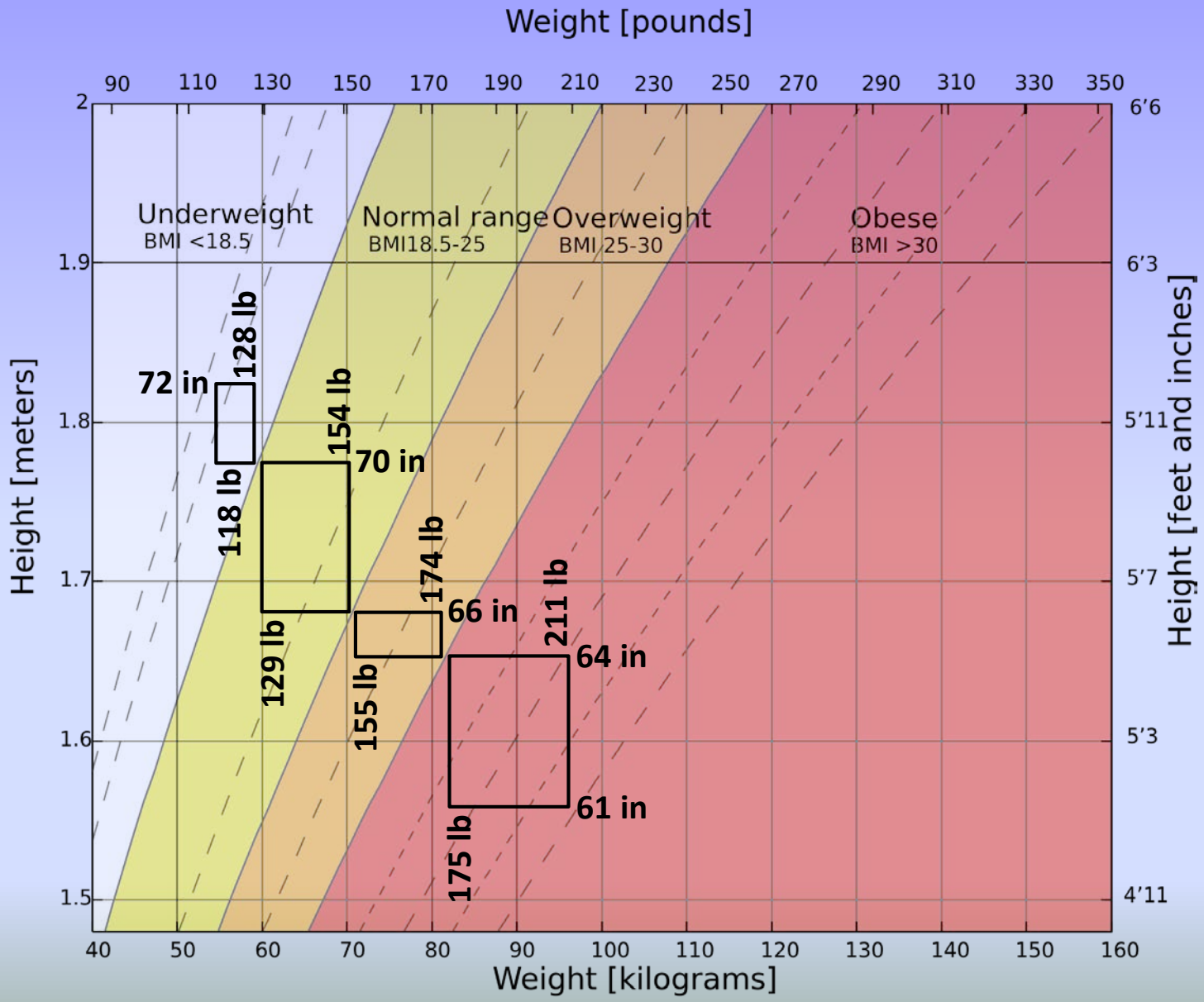
Boundary cases:

Medicare: lower Age

Adult: lower Age

Adult, underweight: upper BMI

Adult report classes and boundaries



Testers have choices

Equivalence class factor values in a partition:	multiple	1
Number of partitions:	1	multiple
Test target:	large	smaller
Dependence on oracle:	some	more
Freedom to test equivalence class boundaries:	limited	yes
Strength to reach equivalence class:	ℓ	any
Nondeterminant strength ($s \leq n + 1$):	$t - \ell + 1$	$t + 1$